Introduction to matrix-product states and algorithms

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1 Convergence of DMRG and TEBD

Use the notebooks groundstate_dmr and groundstate_tebd to obtain the ground-state energies for the transverse field Ising model. Compare the speed of convergence for different parameters $g/J$ on small clusters (e.g., length $L = 8$ and bond dimension $\chi = 8$). The exact energies can be obtained using the ed_ising.py script. Plot the energy differences as function of the number of iteration steps for the two algorithms. How do they compare? How does the finite time step in TEBD affect the energies?

2 Area Law and log-corrections

Use the notebook groundstate_dmr to study the entanglement scaling as function of $L$.

1. Plot the half-chain entanglement entropy $S$ for the middle bond as function of length $L$ for different parameters $g/J$. A divergence of $S$ allows to detect the critical point!

2. Tune to the critical point of the Ising model ($g/J = 1$) and estimate the central charge $c$ of the critical point using the relation $S = \frac{c}{6} \log L$. Carefully check the dependence on the bond dimension $\chi$!
3 PHASE DIAGRAM OF THE ISING MODEL

Use the notebook `groundstate_idmrg` to study the phase diagram of the transverse field Ising model in the thermodynamic limit.

1. Plot the magnetization $m$ and entanglement entropy $S$ as function of the transverse field $g/J$. Determine the phase diagram!

2. Carefully check the convergence of the ground state energy compared to the exact solution for different parameters (e.g., transverse field $g/J$, bond dimension $\chi$, and number of iterations $N$). Examine the continuity of the order parameter $m$ at the critical point for small bond dimensions $\chi$.

3. Tune to the critical point of the Ising model ($g/J = 1$) and estimate the central charge $c$ of the critical point using the relation $S = \frac{c}{6} \log \xi$.

4. Add a symmetry breaking longitudinal field $\hbar \sigma_z$ to the hamiltonian (edit the matrix-product operator). What happens to the critical point?

4 PHASE DIAGRAM OF THE SPIN-1 CHAIN

Use the notebook `groundstate_idmrg` to study the phase diagram of the spin-1 Heisenberg chain with single ion anisotropy

$$H = J \sum_j \mathbf{S}_j \cdot \mathbf{S}_{j+1} + D \sum_j (S_j^z)^2,$$

with $J > 0$.

1. Implement the Hamiltonian as matrix-product operator into the code!

2. Plot the entanglement entropy $S$ as function of $D/J = -2\ldots2$ to determine where phase transitions occur.

3. Plot the staggered magnetization as function of $D/J$.

4. Plot the “topological” order parameter

$$\frac{1}{\chi} \text{Tr}(U_x U_z U_x^\dagger U_z^\dagger)$$